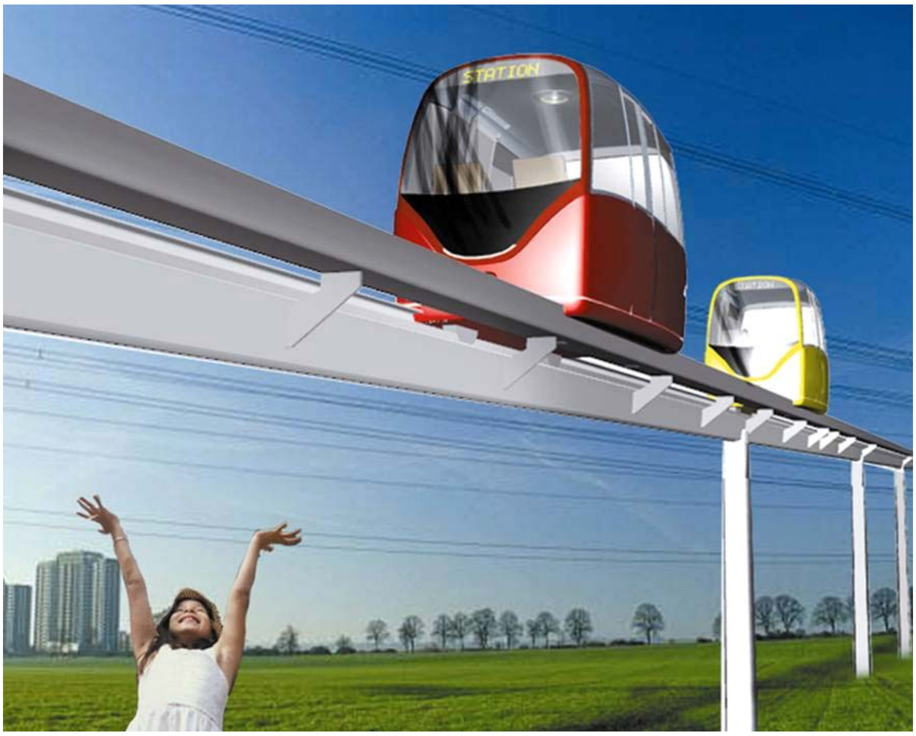
The PRT Project
Phase 1 Design & Engineering

Structural Analysis for PRT



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Analysis Report

CONTENT

1 Summary

Basic structural properties of 7 beam profiles and 6 column profiles have been established using both hand calculations and FEA. Materials of consideration have been steel and pre-stressed concrete.

The structural characteristics of the cross sections evaluated give comparable information on the properties and will be useful for choosing cross sections for the beams and columns of the PRT Rail System.

2 Introduction

The purpose of this document is to present basic structural characteristics for a number of beam and column profiles for the PRT project. The intention is to aid product designers with the choice of profiles, and to assist the project members with their choice of material and cross sections.

Various properties of the beams and structures will be calculated using hand calculations of cross section stress, safety factor with respect to yield, deflection and natural frequencies. Finite Element Analyses (FEA) have been executed to verify the results.

For the columns the properties of stress and critical buckling load will be decided.

3 Formulas

Beams

Bending Moment

$$M = \frac{FL}{8}$$

Max Section Stress

$$\sigma = \frac{Mh}{2I}$$

Deflection

$$y_{\max} = \frac{FL^3}{192EI} + \frac{qL^4}{384EI}$$

Natural Frequency

$$f_1 = \frac{13,86}{2\pi}\sqrt{\frac{EIg}{FL^3 + 0,383qL^4}}$$

Columns

Stress

$$\sigma = \frac{F + qL}{A}$$

Critical Buckling Load

$$F_c = \frac{\pi^2 EI}{4h^2}$$

4 System description

The PRT Rail System consists of a number of horizontal beams supported by vertical columns. The beams carry rails, on which the passenger cars drive.

The calculations were based upon a series of assumptions, listed below:

- Both ends of the beams are rigidly fixed
- The span between the supports of a beam is 20 meters
- The mass of one car is 1000 kgs
- The mass of one car can be seen as a concentrated load
- All the loads on the columns act in a straight vertical direction

4.1 Materials

The properties used for the materials are listed in the table below:

Property	Steel 304	Concrete
Modulus of Elasticity [GPa]	200	8
Tensile Strength, Yield [MPa]	210	100
Density [kg/m^2]	7 800	2 400

5 Analysis approach

General

Every cross section was modeled in the CAD application Ideas-9, and the cross sectional areas and the moments of inertias were calculated using Ideas. The structural characteristics of each section were calculated using the formulas listed above, applied in an Excel workbook.

All together 5 beam cross sections made of concrete and 2 cross sections made of steel were evaluated. For the columns a set of 3 concrete cross sections and 3 steel cross sections were investigated.

5.1 Beams Cross Sections

The 5 concrete beam cross sections are denoted B1 – B5, and the 2 steel cross sections are denoted S1 and S2. See attachment A for schematic drawings of these sections.

To make it possible to compare the results from each cross section, it was necessary to put a global limitation on the profiles. Therefore all profiles will fit inside a circle with diameter 400mm. This way the results give a ranking of beams of approximately the same visual size.

Calculations

The properties were calculated with the following conditions applied:

- One car was positioned equally distanced from the supports of the beam (on the middle of the beam)
- The weight of the beam was equally distributed along the length of the beam

The following properties were found:

Beams	Cross Sectional Area	Density	Weight pr length	Force pr length	Profile height	Moment of Inertia
	m^2	kg/m^3	kg/m	N/m	m	m^4
Concrete						
B1	0,0661	2 400	158,720112	1557,044299	0,332	0,000507263
B2	0,0764	2 400	183,295608	1798,129914	0,305	0,000586046
B3	0,0893	2 400	214,286496	2102,150526	0,33	0,000704725
B4	0,1015	2 400	243,52848	2389,014389	0,4	0,00094053
B5	0,0674	2 400	161,8038	1587,295278	0,33	0,000567744
Steel						
S1	0,0096	7 800	74,568	731,51208	0,33	0,00015752
S2	0,0123	7 800	95,567238	937,5146048	0,4	0,000233098

5.2 Column Cross Sections

All together 3 concrete columns and 3 steel columns were investigated. The concrete columns are denoted B11, B12 and B13, and the steel columns are denoted S11, S12 and S13. See attachment A for schematic drawings of the cross sections.

Again a global limitation was applied, in order to make it possible to compare the results from the calculations. For the columns a 400 mm x 400 mm square made the outer boundaries for the cross sectional area.

Calculations

The stress in the columns was calculated with two different loads – a) 1 car on the rails, and b) 10 cars on the rails.

The total weight down on the column was a load combined from the mass of one beam, the rails (4 units) on one beam – a total of 1062 kgs, and 4 ribs to support the rails – a total of 2000 kgs. Two scenarios were investigated, one with a concrete beam section and the other with a steel beam section. The steel section was calculated to have a mass of 1900 kgs, making a total mass of 5962 kgs work on the columns. The concrete column was calculated to have a mass of 4860 kgs, making a total of 8922 kgs working on the columns.

The properties were calculated with the following assumptions applied:

- One car was positioned right on top of the column
- Only vertical forces are acting on the columns
- The columns are vertically oriented

The following properties were found:

Columns	Cross Sectional Area	Density	Weight pr length	Moment of Inertia
	m^2	kg/m^3	kg/m	m^4
Concrete				
B11	0,1586	2 400	380,56	0,002080023
B12	0,0794	2 400	190,59	0,000261456
B13	0,0592	2 400	141,98	0,000131192
Steel				
S11	0,0087	7 800	67,96	0,00021449
S12	0,0176	7 800	136,89	0,000160104
S13	0,0231	7 800	180,18	0,000571533

6 Results

The following properties were found for the beams:

Beam	Max Section Stress	Safety Factor - Yield	Deflection	Natural Frequency	General Stiffness	Weight/meter
	MPa		mm	Hz	Nm^2	kg/m
Concrete						
B1	25,01	4,00	260,6	1,06	4,06	158,72
B2	21,98	4,55	247,0	1,09	4,69	183,30
B3	22,15	4,52	227,9	1,14	5,64	214,29
B4	22,15	4,51	186,6	1,26	7,52	243,53
B5	22,50	4,44	235,6	1,11	4,54	161,80
Steel						
S1	51,23	4,10	22,6	3,49	31,50	74,57
S2	47,86	4,39	17,1	4,05	46,62	95,57

The following properties were found for the columns:

Column	Stress with concrete beam and one car	Stress with concrete beam and 10 cars	Stress with steel beam and one car	Stress with steel beam and 10 cars	Critical buckling load	Critical weight
	Mpa	Mpa	Mpa	Mpa	N	kg
Concrete						
B11	0,55	1,11	0,37	0,93	1 357 289,53	138 358
B12	1,10	2,21	0,74	1,85	170 609,60	17 391
B13	1,48	2,97	0,99	2,48	85 607,48	8 727
Steel						0
S11	10,05	20,18	6,71	16,85	139 962,61	14 267
S12	4,99	10,02	3,33	8,36	104 473,60	10 650
S13	3,79	7,61	2,53	6,35	372 945,43	38 017

The results of the FE-Analysis is given in Attachment C.

7 Conclusions

The hand calculated values show good correspondence with the results from the FEA.

7.1 Concrete beams

- The cross section with smallest stress is B2 (and highest safety factor against yield)
- The cross section with smallest deflection is B4
- The natural frequencies of the sections are all in the same range, just above 1 Hz
- The lightest cross section is B5

7.2 Steel beams

- The circular cross section (S2) is (not surprisingly) the strongest, has the smallest deflections and highest natural frequency
- The properties of the S1 cross section can be manipulated when varying the wall thickness of the profile

7.3 Concrete columns

- The stress in the columns is far from critical, even with 10 cars on the rails
- The critical buckling load for B3 is approximately half of the critical load for B2

7.4 Steel columns

- The stress in the steel columns is much higher than in the concrete columns, but still far from critical value
- The buckling load of S13 is much higher than the buckling load for the other columns

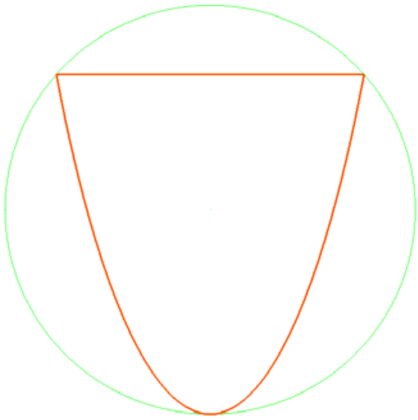
8 Comments

The choice between concrete and steel structures is dependent on a number of aspects. Some of them are listed below:

- Concrete needs less maintenance than steel, and costs on long terms will probably be lower using concrete
- In this type of construction the deflection of the beams will usually be a critical design factor. Normally concrete has larger deformations than steel, but the use of reinforcement and pre-stressed concrete elements can improve this a lot. With a good design it could be possible to make concrete elements less flexible than steel.
- The size and weight of the elements must be considered in regard to limitations in the assembly/building area (like access to site etc).

A more thorough investigation will be needed before a final conclusion can be made. Still the results in this report give a good guideline for superior choice of materials and cross-sections. It will also give good guidance to industrial designers.

9 Attachment A – Beam Cross Sections



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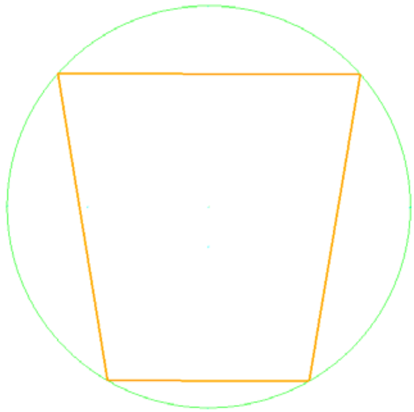

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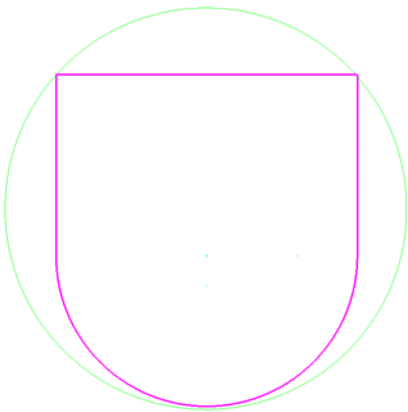

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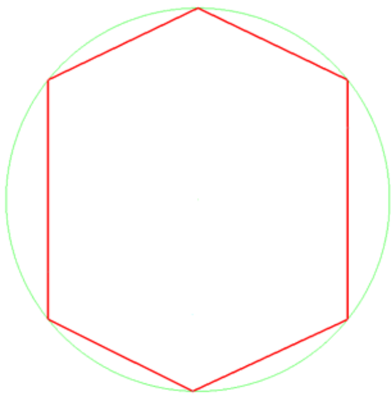

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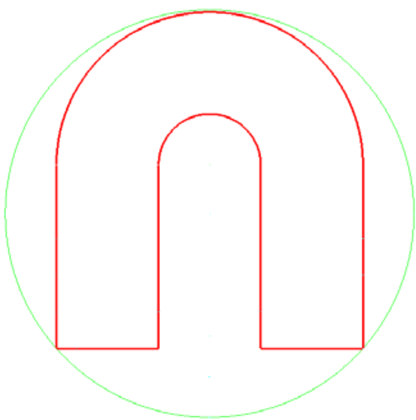

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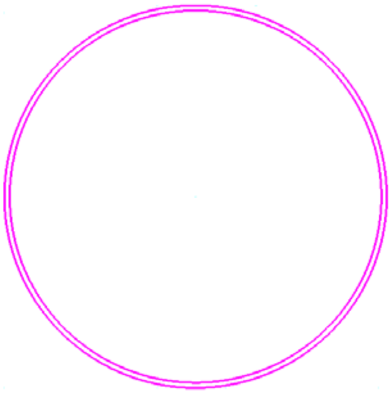

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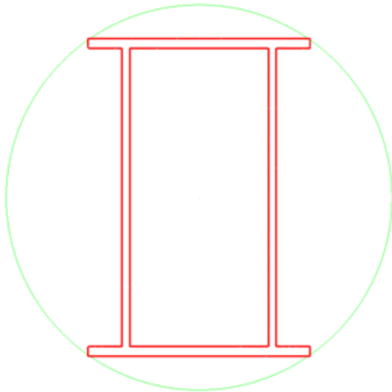
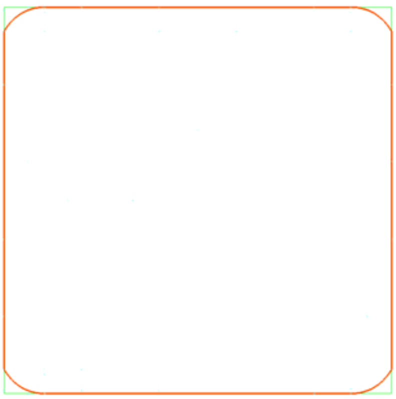


S2

10 Attachment B – Column Cross Sections



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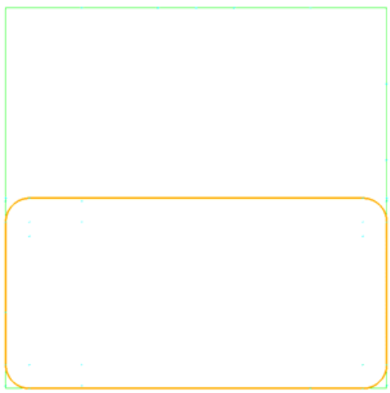

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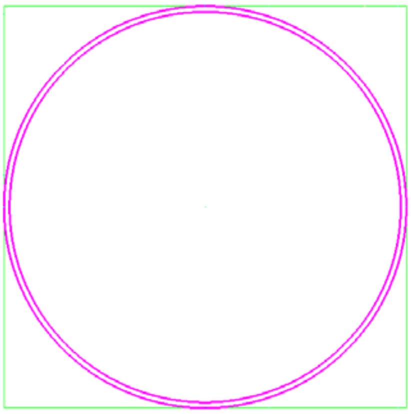

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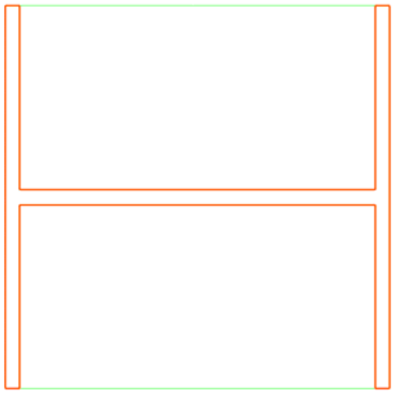

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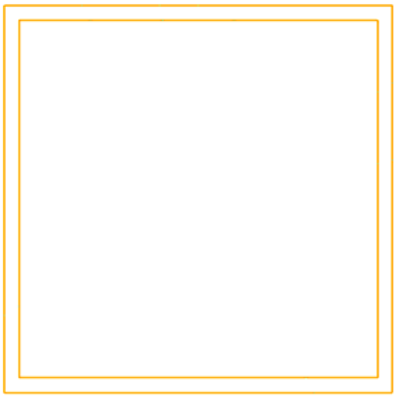
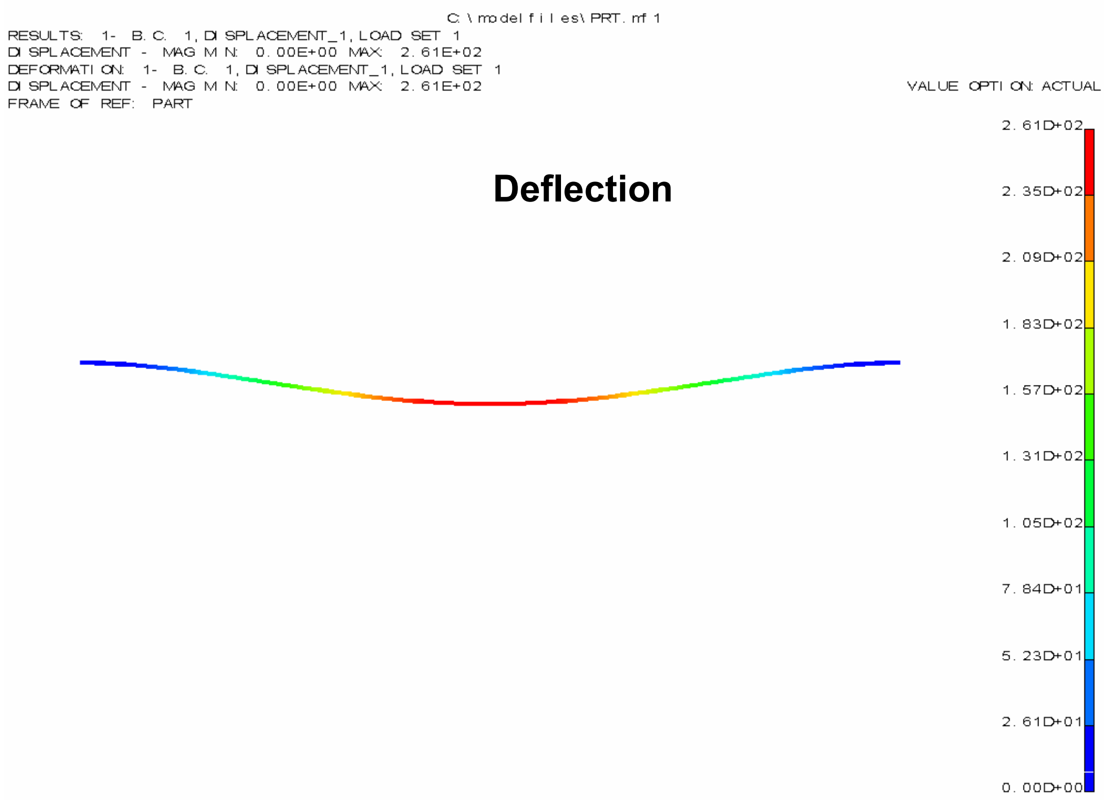
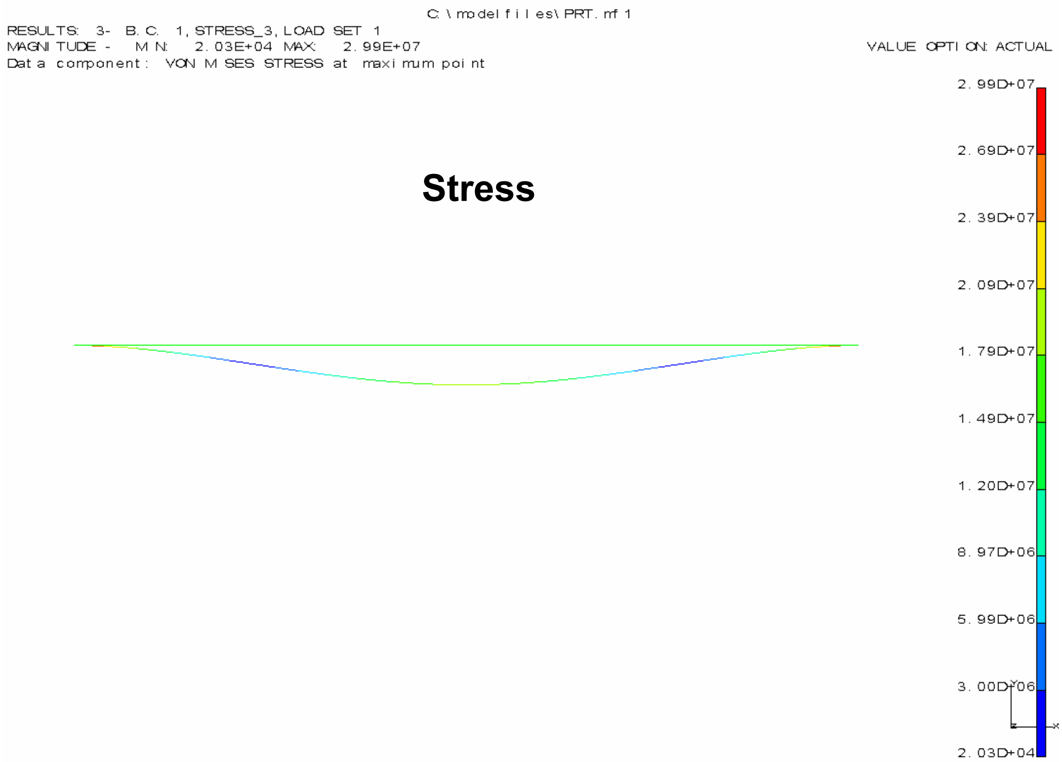


S13

11 Attachment C – Screen dumps of FEA analysis

Beam B1





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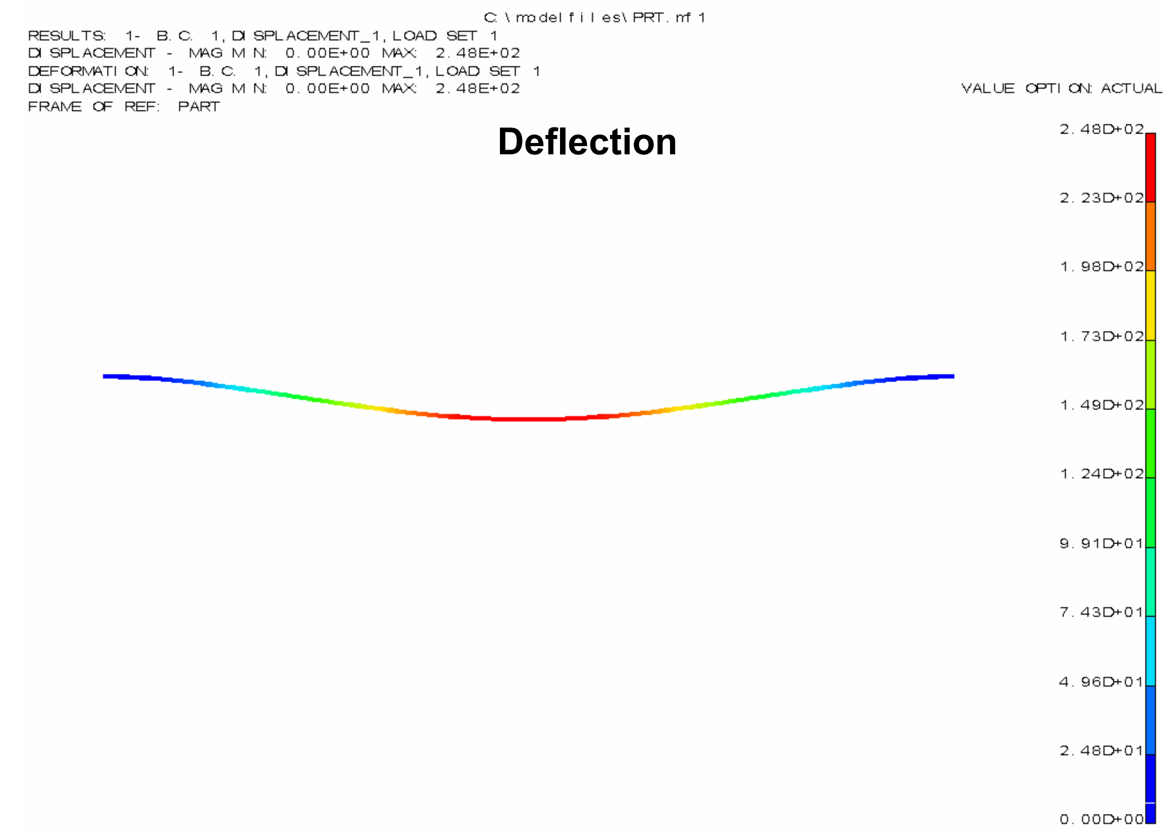
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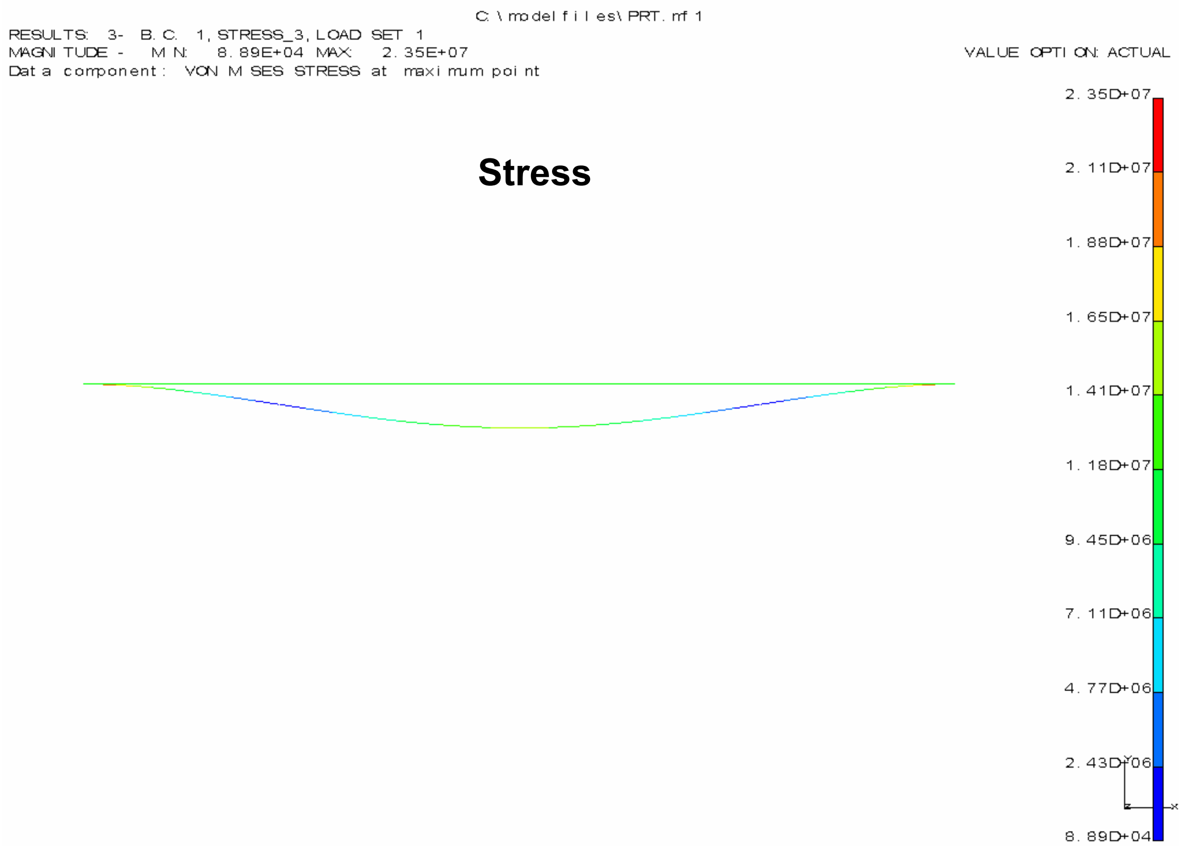
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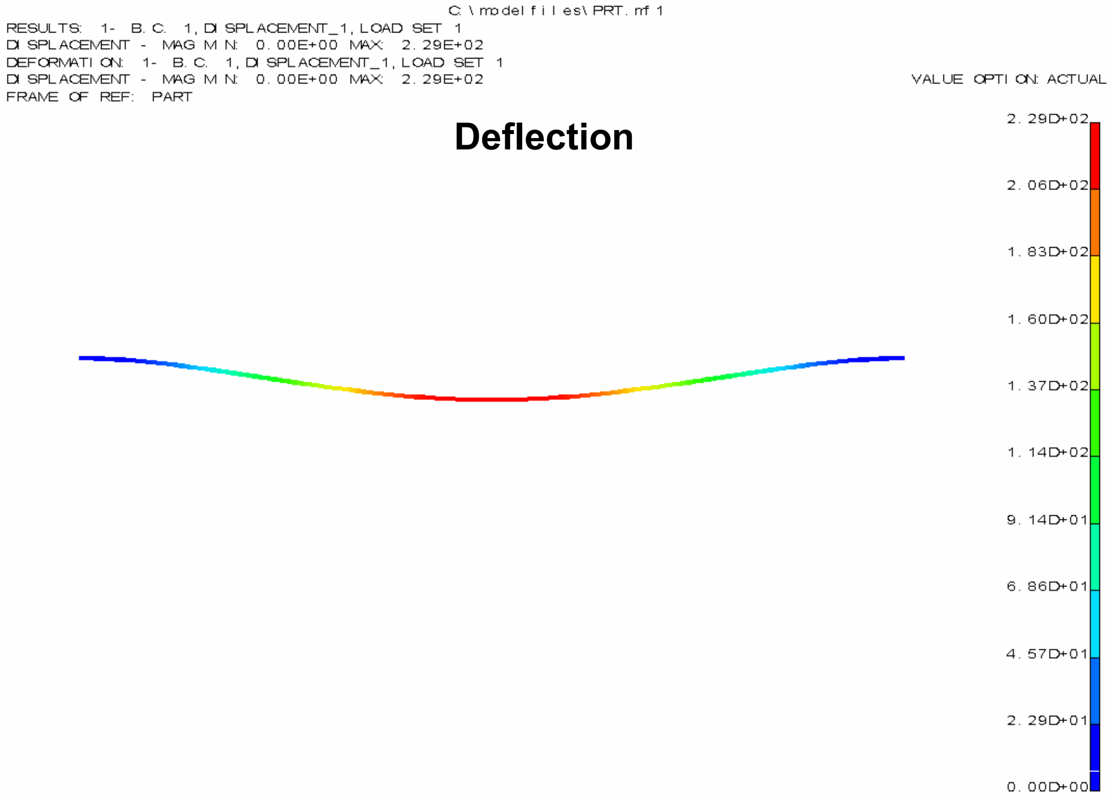
Beam B3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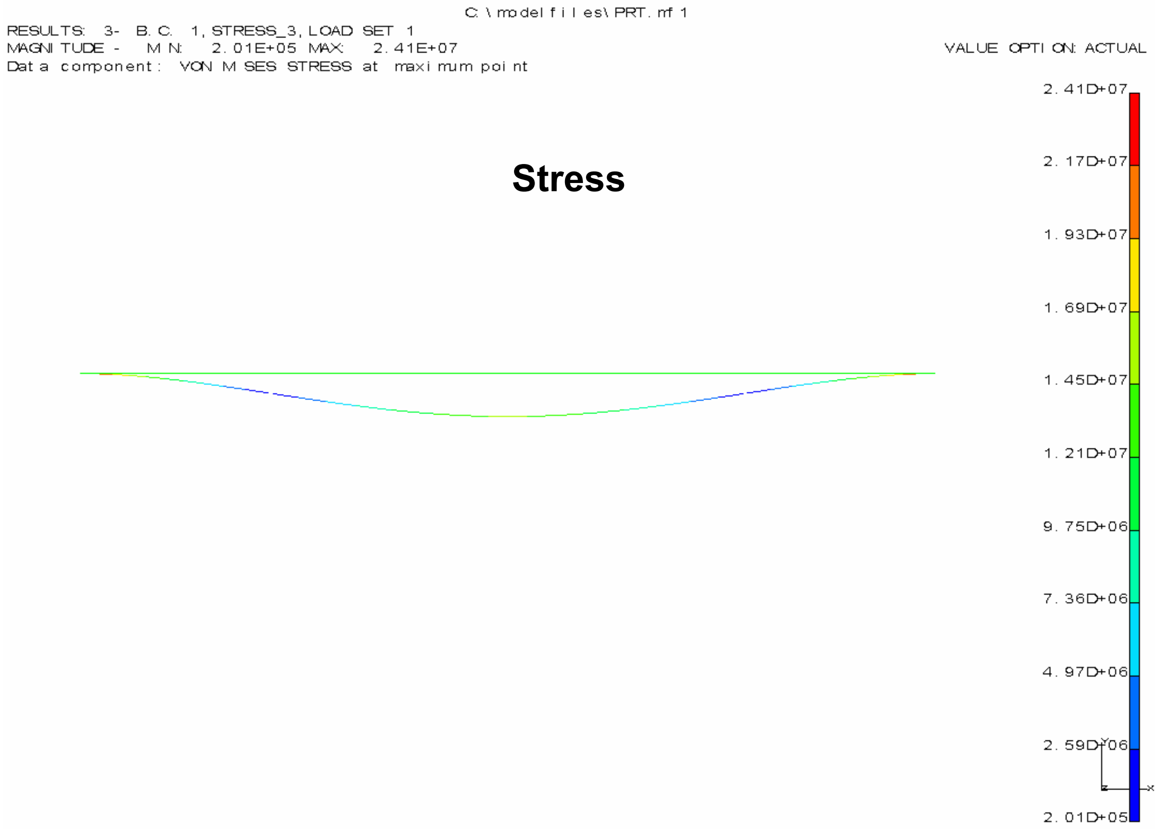
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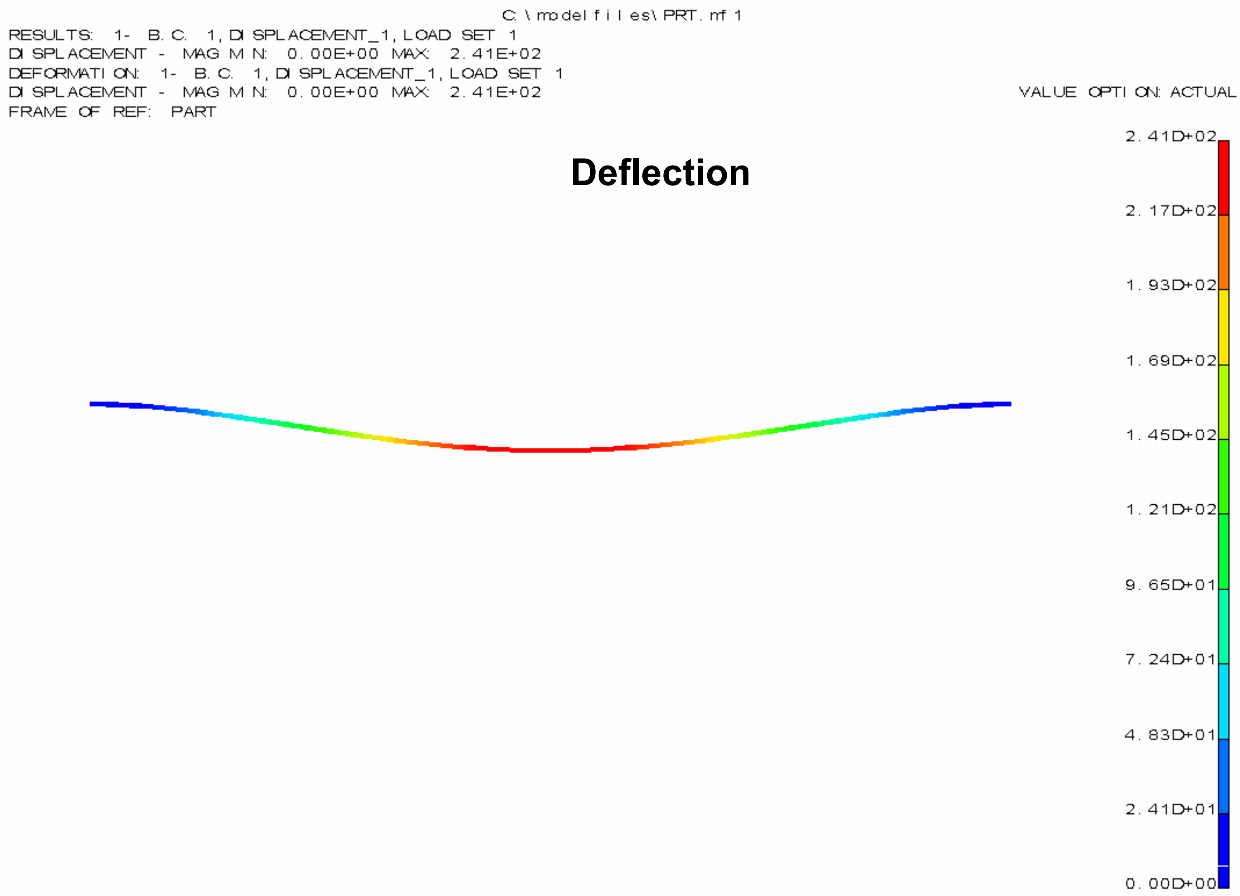
Beam B4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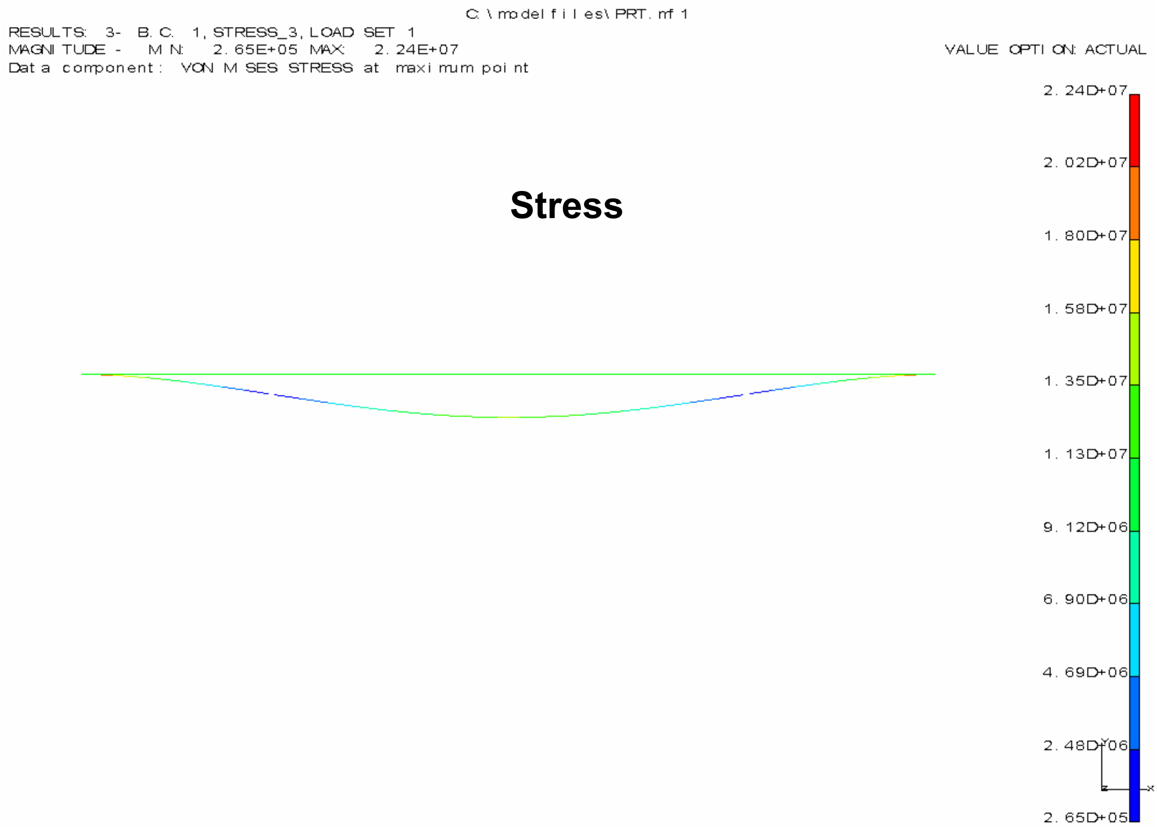
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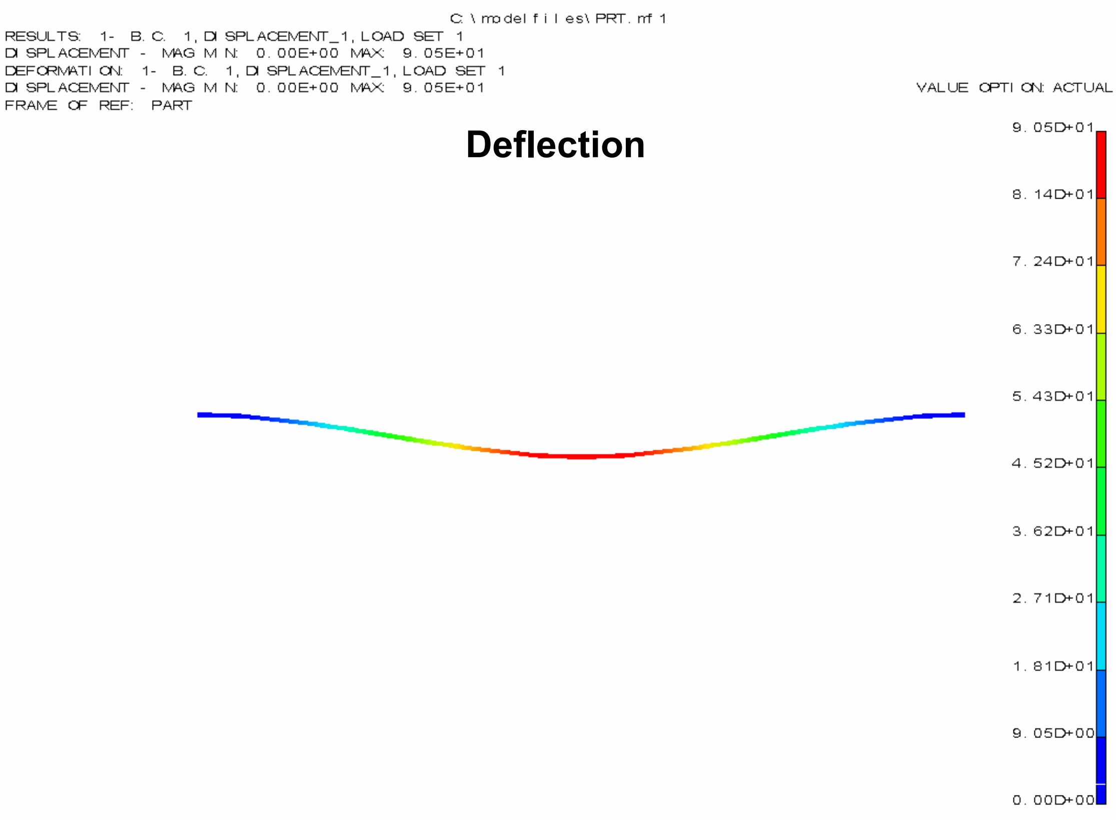
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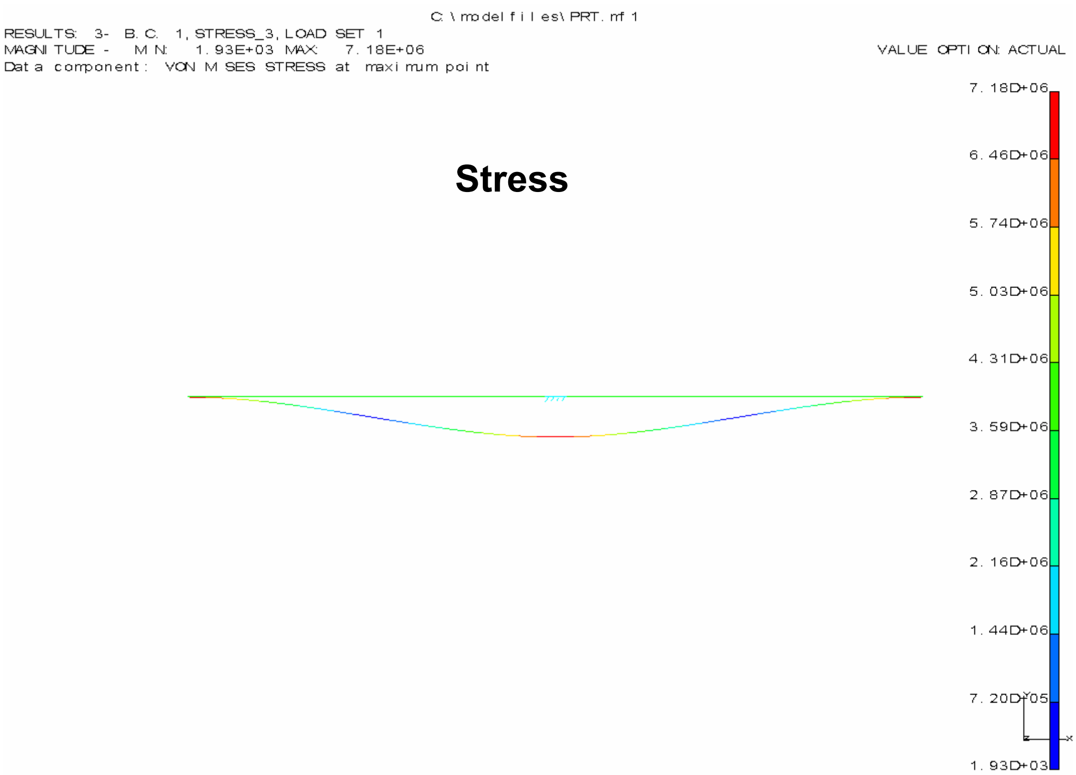
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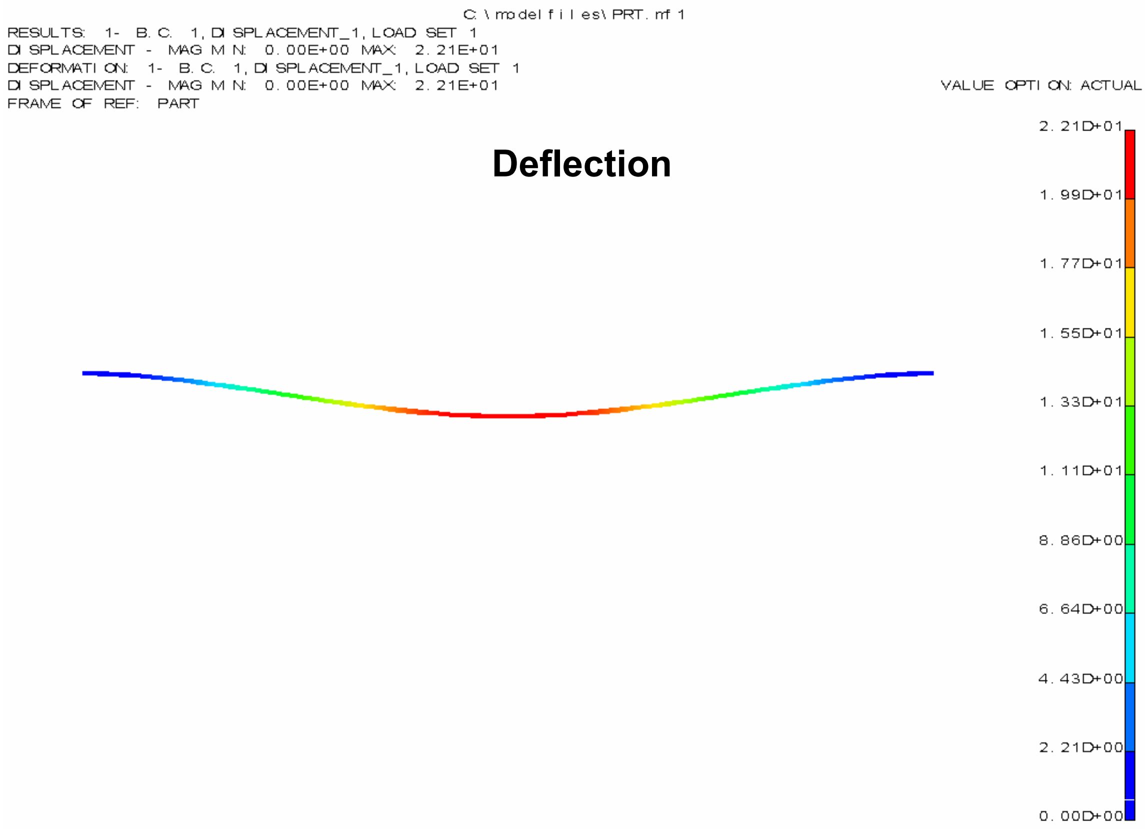
Beam S1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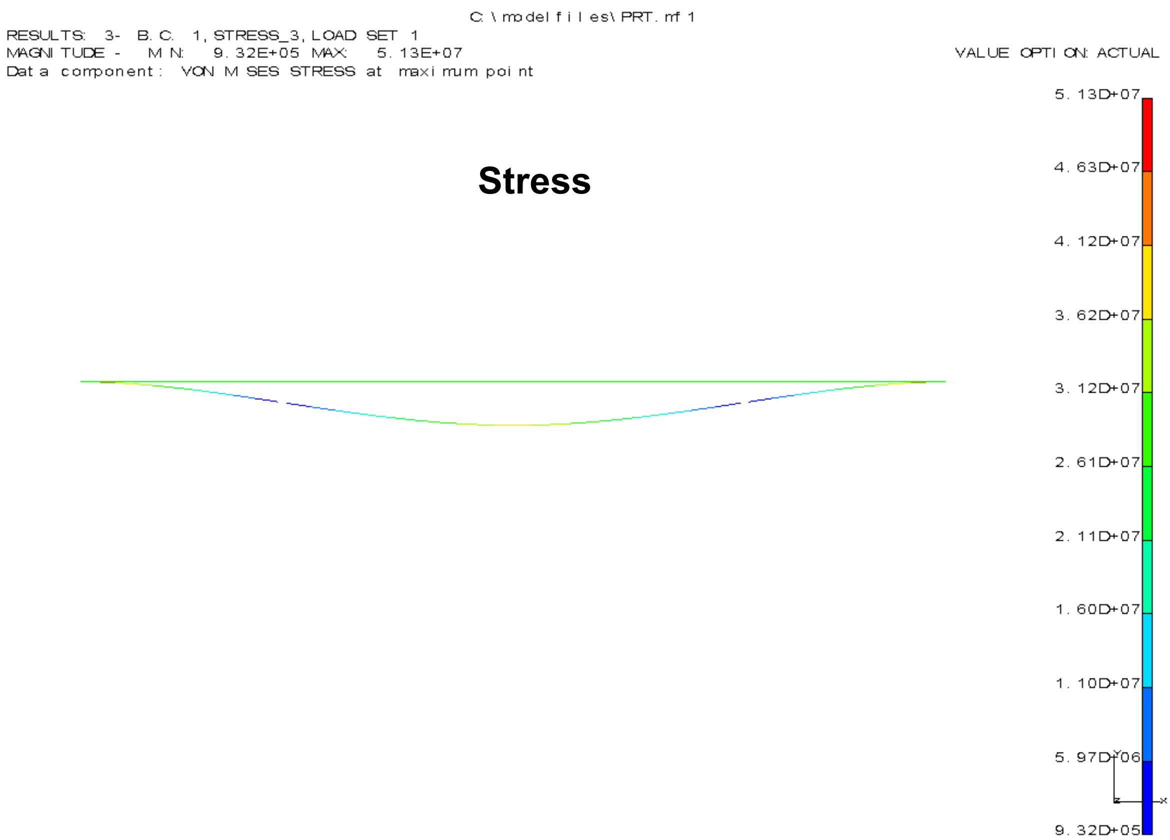
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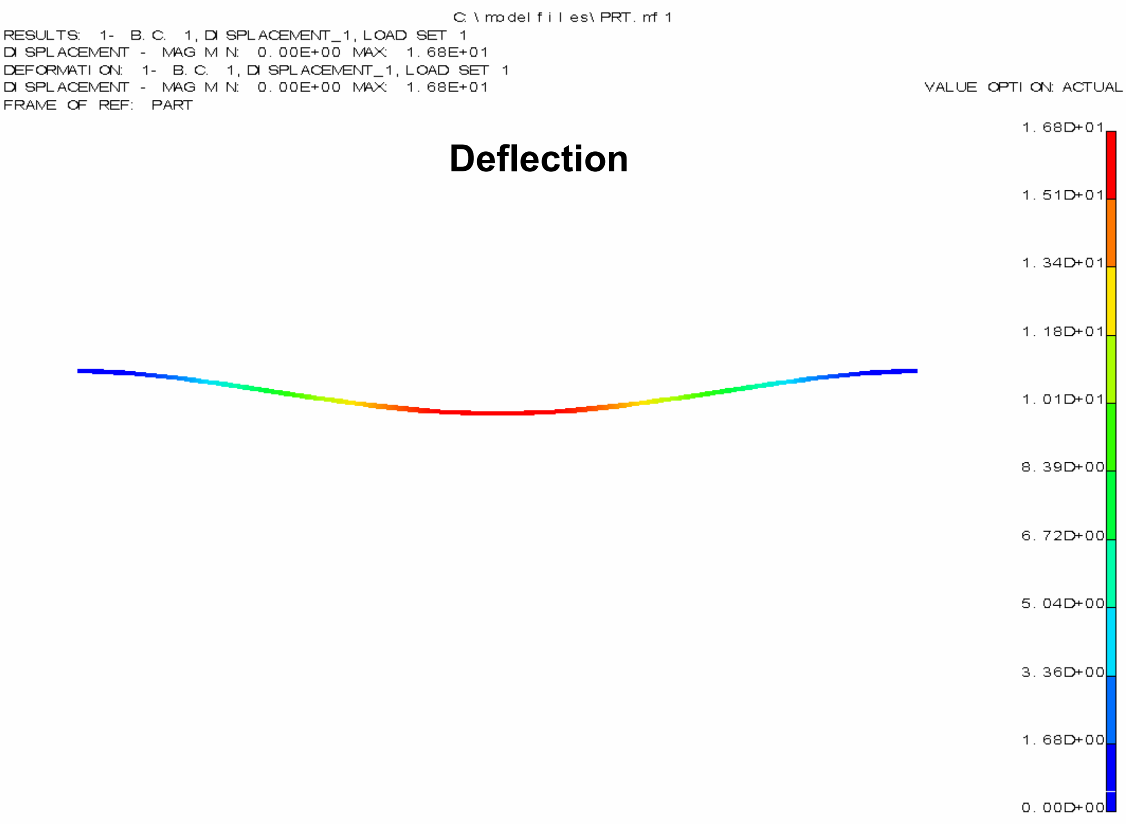
Beam S2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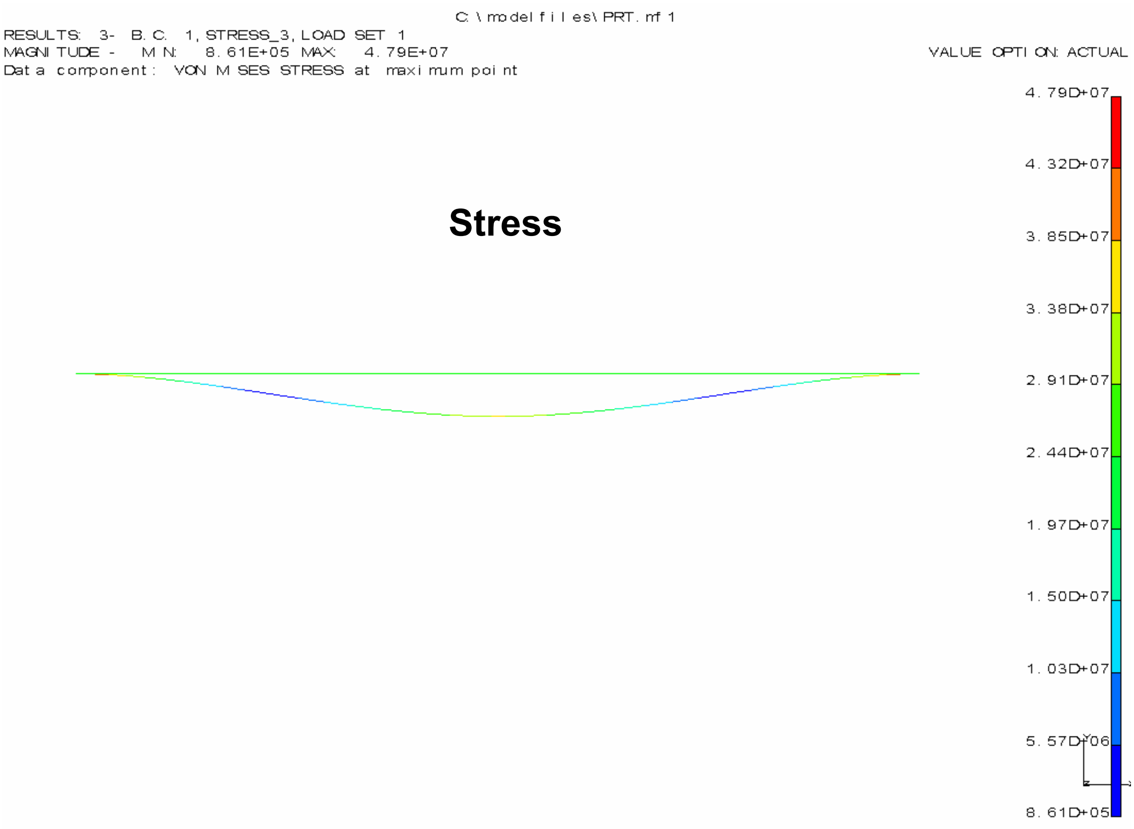
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